Is fake news consistent with the truth?

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1   Introduction

Since the election of Donald Trump as the president of the United States of America, the word "fake news" has become famous and has found kind of a social status, becoming a subject of concern around the world. I will not take part in the discussion about who releases fake news or not, but the goal of this paper is to give a mathematical definition of what a fake news is. Once this is done, the next problem is to see if, as many research and development programs aim at, it is possible to find a parade to fake news. The answer is no, there is no real parade in some sense, but there is even worse: fake news can be obtained just telling the truth.

2   The landscape

What triggered the writing of this paper was the reading of a paper written by Erwan Lamy, associate Professor at ESCP Europe[1], who, referring to a declaration of Reporters Sans Frontières[2], complains about the fact that for RSF, freedom of opinion is guaranteed by the free exchange of ideas and information based on facts[3]. Even if Lamy is more than convincing in his paper, this is not the subject which will be tackled here, the question is more to know what we call information, since Lamy challenges RSF's definition.

On the other hand, we have, since a long time now, an example of kind of fake news which has been treated mathematically in the field of cryptology and which seems to have been forgotten if ever known by the protagonists of today's discussions. This is perfectly well described in Simmons's paper whose title is *the prisoners' problem and the subliminal channel* [1]. Basically, in any communication channel, there can be a subliminal one whose purpose is to give extra hidden information. Obviously, for the one who only gets access to the non-subliminal channel, this can be considered as fake news.

The question therefore is to know if we can give a more general result and reach it without calling for the cryptologic background which might look a bit complicated. I achieve this only through reasoning in an easy mathematical framework, with no calculation.

3   What is information?

Before speaking about fake news, we must give a definition of what information is. For RSF, as we saw above, it seems to be facts. Is it? As we shall see soon, it is not. But before saying what information is, we must describe its scope. And this is where the breakthrough occurs. First, information is only worth in the human context and it therefore refers only to human activity. What I mean here is that whatever the fact, it is interesting as long as it concerns our "environment". This word is into quotes because if covers all what you can think about, from material to intellectual, existing, past or to come and so on about man and its global environment (without quote that time). And, of course, our environment changes or can change, so we shall note it as a random vector $\left(Y_1(t),...,Y_n(t),...\right)$. This vector can be infinite the case being and t represents time. Now, informing will consist in reporting some facts and giving the picture of the potentially changed vector $\left(Y_1(t_1),...,Y_n(t_1),...\right), t_1 > t$. The "news reader",

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can check the facts if we consider RSF's approach and measure what happened though calculating $Y(t_1) - Y(t)$, where Y stands for the whole vector. And we could stay at this point. This reminds me my mathematics teacher when I was in high school and who used to say: "do you know the story of the Mongolian who said, stone, stone, stone…?" Nobody dared to ask him what he meant, but I finally went to him after some years. He told me that the Mongolian who receives a stone on his head does not necessarily know the cause why he received it, and says, "stone, stone, stone". And he complained that the students invoked theorems of mathematics as their justification in the same way, such as "Fermat, Fermat, Fermat", encouraging them to make logical arguments instead. This is exactly the same story for information. The description of facts is necessary but is not enough. What is the most interesting in the information is to find the reason why such events occurred. And once you have a clue about why, you can build your opinion on it.

Let us go a bit deeper into mathematics. The transformation of the environment vector can be written as

$$\left(Y_1(t_1), ..., Y_n(t_1), ... \right) = f\left(\left(Y(t), ..., Y_n(t), ... \right) \right)$$

Where f must be a "logical" function[4]. So, information should consist in both giving the resulting environment vector and the logical function.

4   A first problem

If you start from a given situation and get a new one, the big problem you face is that you can have plenty of functions f which will fit the equation above and which are "logical". So, how can you choose the right one?

In fact, if we try to better understand how the human society works, there are civilizations. What is a civilization? This is a common way to share the understanding of the environment. Therefore, this is going to restrict, in the problem just mentioned, the number of possibilities for the functions f. Typically, for a given civilization, this will consist in having a restricted family of functions[5], say, $F = \{ f_\alpha, \alpha \in I \}$ where α is a parameter and I is a set. So, for a given fact, the explanation of a given civilization will consist in picking out one function f_α as the logical cause of the events which occurred. This function f_α not only covers the purpose of giving a logical explanation, it also has the purpose of "closing" the set F, that is, showing if not proving that the foundations of the civilization allow tackling and solving the problem in a meaningful and "positive" way.

Now, let us come back to our initial problem. How to choose, given the two environment vectors $\left(Y_1(t), ..., Y_n(t), ... \right)$ and $\left(Y_1(t_1), ..., Y_n(t_1), ... \right)$ the "right" function f_α, since there are many which fit? Obviously, paraphrasing Arrow and his famous theorem (which brought him to be awarded the Nobel Prize of economics), the choice is dictatorial. So, we point out, because we did not make any assumption about the regime in place, that informing, whatever the society, is dictatorial. Those claiming for democracy and the values of democracy should lower down their criticisms towards dictatorships given what we just noticed…

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[4] It is probably worth here to warn the reader. Logical does not mean, strictly speaking, mathematical logic even if mathematical logic is at stake. Indeed, logical here means in the societal or cultural context, which can imply religion, law and so on.

[5] These functions are implicitly or intuitively known by the population. This is what means that some is a representative of a civilization. On the other hand, it is trivial that any migrant coming from any alien civilization, will not be in such a situation and can understand the news differently…

I shall come back on the comparison between democracies and so-called dictatorships later.

5 Fake news

5.1 Trivial fake news

The trivial fake news is when what is published is not the truth. Typically, assuming the initial vector was known, that is, $\left(Y_1(t),...,Y_n(t),...\right)$, you publish some false new environment vector, $\left(Y_1^{'}(t_1),...,Y_n^{'}(t_1),...\right)$ which is different from the true one which is $\left(Y_1(t_1),...,Y_n(t_1),...\right)$. So, you deliberately lie. Such kind of fake news, whereas it exists, is not really interesting because it is too coarse. I therefore shall not insist on this, but we all know that such kind of fake news was at stake, such as the incubators in Iraq for the First Gulf War, the weapons of mass destruction in Iraq for the Second Gulf War, some false information in Romania in the 1990s and so on. You will notice that this is the only case thought about by RSF, which proves the poor intellectual level within this organization whose profession is, nevertheless, information.

5.2 Manipulation of the opinion

It can be bothering, when you govern your country, to choose the "right" function f_α because it could compromise some interests. Therefore, it is often, if not always, tempting to shorten the environment vector into a smaller finite number of components. Typically, you only consider $\left(Y_1(t),...,Y_m(t)\right)$ with m small. This has the advantage of increasing, within F, the number of functions f_α you can choose[6] and then improving your dictatorial choice. Such a way of doing things is known from the Catholics since the beginning. This is what they call lying by omission. And it is a fact that such fake news is very common in all societies. Just please notice that in this case, you have told the truth about the events which occurred.

5.3 The fake news

What we have described was, say, in a national context. Now, globalization has occurred and the means to broadcast news are no more local so that information is global. CNN, RT, France 24, BBC and many others release news 24/7 in the whole world. And here is how these organizations can release "fake news" strictly telling the truth. First, they can counter the two preceding ways of manipulating people we saw above. If the released environment vector is false, it is then straightforward to tell the true one and prove it. This will be considered as fake news in the adversary referential, but, at least, this case, once again, is too coarse to be really considered, even if this is the preferred one which is used in war time unfortunately[7].

The second way to act, is to extend the shortened environment vector in order to "kill" the initial choice of the function f_α. The reader should be obviously convinced that this is an easy attack. Indeed, if you are obliged to shorten the environment vector in order to find the "good" function f_α, this means this function is "fragile" and can be dismissed with very few additional parameters. Fragile, here, mathematically means that the number of additional "well suited" f_α you could get from the reduction of the length of the environment vector, is small, which gives a great probability to break it through extension.

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[6] Because this decreases the constraints on the choice.
[7] Essentially because verifying the truth in a conflict zone is difficult.

Let us see this mathematically. Because of laziness (e.g. trying to decrease the cost) or manipulation will, most media if not all, voluntarily, as we saw above, shorten the environment vector into $(Y_1,...,Y_m)$ with m small. Now, they therefore inform according to

$$(Y_1(t_1),...,Y_m(t_1)) = f_\alpha((Y_1(t),...,Y_m(t))), f_\alpha \in F$$

If you want to counter this, staying in the same civilization referential, what do you do? A possible way is the following. Pick $k > m$ and inform according to

$$(Y_1(t_1),...,Y_m(t_1),...,Y_k(t_1)) = f_\beta((Y_1(t),...,Y_m(t),...,Y_k(t))), f_\beta \in F, f_\beta \neq f_\alpha$$

Clearly, this is a bombshell! You strictly respect the truth, you give even "more truth" than the others, you stay in the civilization context, but you are going to be suspect of releasing fake news, as we can see this happening every day.

5.4   The shock of civilizations

Let us now turn to the deep fight. Imagine we face two "competitors" from different civilizations. The set of functions for the first one is F whereas it is G for the second. Let us consider the national news which can be represented by the following scheme

$$(Y_1(t_1),...,Y_n(t_1),...) = f_\alpha((Y_1(t),...,Y_n(t),...)), f_\alpha \in F$$

But now, the "adversary" looking at the events through the prism of its civilization will inform according to

$$(Y_1(t_1),...,Y_n(t_1),...) = g_\beta((Y_1(t),...,Y_n(t),...)), g_\beta \in G$$

For sure, while the truth is saved in both cases, both sides will have a trend to consider that the information of the other is kind of a fake news. Of course, I assumed that $g_\beta \neq f_\alpha$, but the complicated case is when $f_\alpha, g_\beta \in F \bigcap G$.

6   Short term information

Until now, I just considered times $t_1 > t$, and I made no assumption on the value of $t_1 - t$. But since several decades we have permanent information media which release permanent change, so that the information process can be described as

$$(Y_1(t+\delta t),...,Y_m(t+\delta t)) = f_{\alpha,\delta t}((Y_1(t),...,Y_m(t))), f_{\alpha,\delta t} \in F$$

Where δt is a very short amount of time, which implies shortening the length of the environment vector, at least for practical reasons if not for better manipulation as we already saw[8]. I emphasized that the logical function will depend on δt because the smaller δt the less "computation time" you have to choose the interpretation function or the less time you have to really think about what happened, which

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[8] Our approach brings us to wonder whether the creation of permanent news channels was not deliberately put in place in order to better manipulate the people. Such creation could have been the result of a mathematical approach as the one I present here.

is even worse[9]. This has the obvious property of making this information very fragile and it can easily be counteracted via the method of the above paragraph.

So, it is not surprising that the notion of fake news took place just now, after the settlement of worldwide media informing people about very short-term events.

Let us look at this more carefully, and let us take, for the sake of simplicity, a constant value δt. We have a recurring information process, as long as we consider the same variables in the environment vector (which is not necessary the case in the true world, of course, but can be justified if we limit ourselves to a given subject) which we can describe as

$$\left(Y_1\left(t+k.\delta t\right),...,Y_m\left(t+k.\delta t\right)\right) = f_{\alpha_{k-1},\delta t}\left(Y_1\left(t+(k-1).\delta t\right),...,Y_m\left(t+(k-1).\delta t\right)\right)$$

For going from the time t to the time $t+n.\delta t$ we have the explanation which is given by the function

$$f = f_{\alpha_0,\delta t} \circ f_{\alpha_1,\delta t} \circ ... \circ f_{\alpha_{n-1},\delta t}$$

In theory, there should be a constraint on f and it is very simple to write it. We simply need coherence, which means that $f \in F$. If the information delivered is "reliable", this can put, under some circumstances, some constraints on the choices of the $f_{\alpha_i,\delta t}$. This is the difference between short-term manipulation and long term. The latter is much more difficult. Therefore, it is much easier to attack through "true fake news" from the outside, by showing for example that in fact $f \notin F$. But in the case of manipulation, limiting the possibilities for f might become difficult and since the manipulators will never insist on the function f itself (they will rather insist on the $f_{\alpha_i,\delta t}$), it can be pretty easy to emphasize what the resulting function f is for an opponent, killing the preceding propaganda, with the arguments of the adversary!

7 History

The point of view of the preceding paragraph also explains why the work of historians is difficult and, to some extent, what it consists in. Indeed, if the time $t_1 - t$ is big, then you have to reconstitute, on the one hand, the function f as we just saw, check that $f \in F$, but if you want to be objective, you should also try to look at the events under the point of view of other civilizations, in particular when there has been wars between civilizations and look at the same events with $g \in G$. Discussing then what we can draw from the comparison of the different approaches should be of most importance to have a clear picture of what history was.

However, there is an additional big problem concerning history. Indeed, the civilization referential is going to change over time. Mathematically, this can be written $F(t)$ to emphasize that the set of admissible functions varies with time. The problem occurs when we face the following situation. We historically explain a period between, say, time t and t_1, the function which explains what happened during this period, is, say $f \in F(t_2), t_1 < t_2 < t_3$, we are at time t_3 with $F(t_3) \neq F(t_2)$ and, of course $f \notin F(t_3)$. Then begins, generally, the beginning of a judgment by the historians of people, with a referential which was not pre-existing at the time of the events. Unfortunately, not only this constantly happens, but it seems to be kind of martingale from historians to use this method to discredit well-known

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[9] The less computation time for the news broadcaster and the less time for the listener to think, could also be a valid interpretation…

respected historical figures who would not deserve such critics if they were "judged" by the criteria of their epoch. We should therefore care about what modern historians say and be very skeptical in many cases when they use obviously in their referential what we usually call the reading grid of today.

8 Democracy

The proposed model here brings us very naturally to define what democracy should be. Most people think that democracy can be summarized by having people vote. But, as far as we know, there are also votes in dictatorships. In Soviet Union for example, there were. So, what can be a democracy under the light of our model?

This is quite simple! In fact, it consists in discussing the content of the set F. The election process should consist in voting on F. Is it what happens, in particular in the countries which claim they are the champions of democracy? Obviously no, be it the USA, the UK, France, Germany and so on. How is it possible to be so sure of this? Simply listening to the debates. This kind of topic is never tackled and, worse, the media do whatever they can to avoid having these deep discussions. For this they define the environment sub-vector of their choice as being the only one which will be discussed such as $\left(Y_1(t),...,Y_n(t)\right)$ and they will take a very short δt looking at what the different candidates of their choice propose on this only. The ones who want to tackle other subjects are disqualified and cannot access the media and unfortunately the people are fooled whereas they are persuaded that they live in a democracy. This is only the method which differs, but in the end, the so-called democracies are true dictatorships.

9 Fighting against fake news

Given what we have seen and easily proved, fighting against fake news is almost impossible since fake news can be consistent with the truth. But what do we see flourishing in western countries today? Simply anti-fake news algorithms, fake news detectors and so on. What can we say about this? Simply that these means are only propaganda!

10 Conclusion

Fake news is a concept which to some extent is relative to a civilization, and some fake news, in the sense that it goes against the interest of the local power, can be released saying absolutely the truth. This is even easier when the massive use of short-term information has the only goal to manipulate the populations. Under such circumstances, all the countries, mainly the western ones, which want to fight against fake news, are in deep distress, since when they complain against fake news, they only complain against their inability to fully manipulate their populations. It is also not complicated to use the approach of this study to find recipes for manipulating information and make fakes news which is true.

11 Acknowledgement

I acknowledge President Donald Trump for having put in the forefront of worries of humankind the phenomenon of fake news. I acknowledge President Vladimir Putin for having accepted to play the role of the bad guy whom everybody wants to bash. I finally acknowledge President Emmanuel Macron, who unsuccessfully tried to get rid of Russia Today and Sputnik media in France, presenting them as fake news broadcasters, which triggered this study of mine, since, reading these media, I could obviously notice that despite the fact that what they said was true, it was qualified of fake news by President Emmanuel Macron.

12 References

[1] G. J. Simmons, «The Prisoners' Problem and the Subliminal Channel,» *Advances in Cryptology,* pp. 51-67, 1984.